MATHEMATICS

Infinite Confusion

Rudy Rucker

David Foster Wallace is a great writer, known for his stories and essays as well as the inspiring novel *Infinite Jest* (*1*). Wallace's work is revelatory, funny, and post-ironic. I fully expected to enjoy *Everything and More*. But it's a train wreck of a book, a disaster. Nonmathematicians will find *Everything and More* unreadable, and mathematicians will view it with, at best, sardonic amusement. Crippling errors abound.

Everything and More
A Compact History of ∞
by David Foster Wallace
Norton, New York, 2003. 331 pp. $23.95, C$36. ISBN 0-393-00338-8. Weidenfeld and Nicolson, London, 2003. £14.99. ISBN 0-297-64567-6.

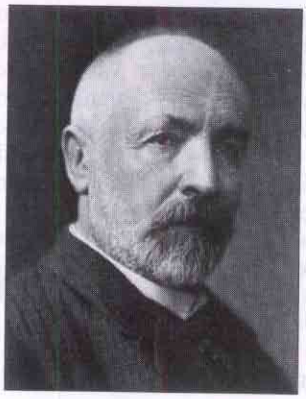
Wallace formats his book as a parody of scholarship. He provides no table of contents, and he titles his sections with numbers, possibly as an homage to the style of 19th-century mathematicians. There are over 400 footnotes squeezed into the book's 305 pages of text. Wallace uses idiosyncratic, pettifogging abbreviations such as "G.C.P.F.S." for "General Convergence Problem of Fourier Series," "IYI" for "If You're Interested, "E.G." for "Emergency Glossary," and "Q.E.I." for "Quick Embedded Interpolation." Here's how he ends one of his subsections:

> END Q.F.-V.-T.I. RETURN TO §7c AT THE ¶ ON p. 256 W/ ASTERISK AT END

Wallace begins the book with some chatty vaporing about how hard it is to think abstractly. He describes his thought processes in an engagingly conversational tone, as comfortable as a letter from a friend. He gets off a fine footnote about the difficulty of being a revolutionary mathematician like Georg Cantor: "Naturally, the significant results that legitimize a mathematical theory take time to derive, and then even more time to be fully accepted, and of course throughout this time the Insanity-v.-Genius question remains undecided, probably even for the mathematician himself, so that he's developing his theory and cooking his proofs under conditions of enormous personal stress and doubt…." And he has a very nice discussion of Zeno and the Greek philosophers.

The trouble begins about one-third of the way through the book. "§3a. It becomes appropriate at this point to bag all pretense of narrative continuity and to whip through several centuries schematically…." Scores of symbol-laden theorems and definitions are inelegantly dumped onto the pages, like a truckload of concrete blocks.

The presentation grows increasingly frantic and disorganized. Nothing makes sense. Wallace's few proof sketches are labored and opaque. He fails to provide illuminating examples, and, worst of all, he botches key mathematical facts. In order to illustrate the subtle nature of infinity in mathematical analysis, for instance, he needs to introduce the notion of uniform convergence. But the definition he provides is incorrect, and he fails to supply the expected example of a nonuniformly convergent sequence of continuous functions that converges to a discontinuous function.



Founder of a mathematics of infinity. On the basis of his set theory, Georg Cantor concluded that a series of transfinite numbers represent different orders of infinity.

When we finally get to the work of Richard Dedekind and Cantor, things are better for a little while. Wallace gives a pellucid explanation of why Dedekind used actually infinite sets of rational numbers to define arbitrary real numbers. But then he quixotically attempts to describe Cantor's early work on the convergence of Fourier series. Wallace's presentation founders upon his inability to give a cogent overview of transfinite ordinal numbers. He makes a hash of the axioms of Zermelo-Fraenkel set theory, stating that the axiom of infinity is a "variant" of the axiom of separation and leaving out the axiom of replacement. He incorrectly suggests that the axiom of choice is used for Cantor's diagonal argument, and so on.

All this might be forgivable if Wallace managed to strike the right tone. In *Out Of My League* and *Paper Lion*, the literary eminence George Plimpton wrote enjoyable accounts of his efforts to play the part of a professional athlete (*2*, *3*). This worked because Plimpton projected self-deprecating humor. But the reader of *Everything and More* encounters hectoring phrases such as "Here's one of those places where it's simply impossible to tell whether or not what's just been said will make sense to a general reader." Elsewhere, the reader

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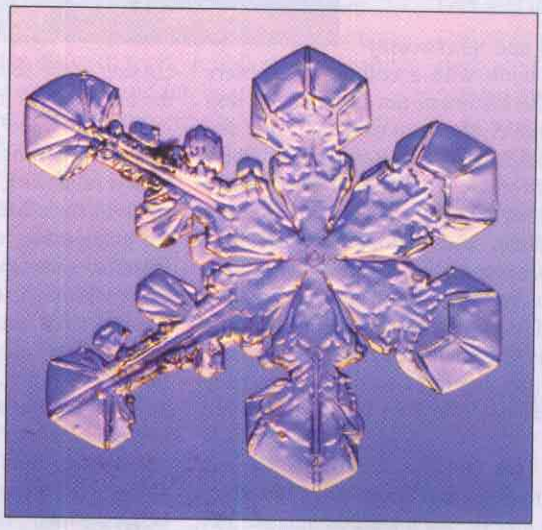

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BROWSINGS

The Snowflake. Winter's Secret Beauty. *Kenneth Libbrecht; photography by Patricia Rasmussen.* Voyageur, Stillwater, MN, 2003. 112 pp. $20. ISBN 0-89658-630-8.

Snow crystals are renowned for the infinite diversity of their shapes. Libbrecht discusses how temperature, humidity, and the processes of branching and faceting (the formation of thin plates and flat edges) interact to create these often complex forms. Rasmussen's striking color microphotographs include this uncharacteristically asymmetric example.

is warned, "It would maybe be good to prepare yourself, emotionally, for having to read the following more than once."

The nadir comes when Wallace tries to write about the continuum problem. He proclaims, "Mathematically speaking, the truth about the Continuum Hypothesis is more complicated than pop writers let on." But he himself seems unable to distinguish between Cantor's continuum problem and Cantor's continuum hypothesis (C. H.). The former asks a question: Where does the cardinality **c** of the continuum lie in the aleph ($\aleph$) hierarchy? The latter proposes an answer: The cardinality c is $\aleph_1$.

And so we arrive at this wretched farrago: "The C.H. gets characterized in all kinds of different ways—'Is the power of the Continuum equivalent to that of the second number class?'; 'Do the real numbers constitute the Power Set of the rational numbers?'; 'Is **c** the same as $2^{\aleph_0}$?'; 'Does $\mathbf{c} = \aleph_1$?'" The first and fourth questions are correct statements of the continuum problem, whereas the second and third questions have simple "yes" answers—for it is elementary to prove that the set of real numbers has the same size as the power set of the countable set of rational numbers, whose cardinality is also known as $2^{\aleph_0}$.

The book closes in a red haze of shame. Wallace doesn't have time to explain the transfinite ordinals after all. As a parting shot, he gives an incorrect characterization of Kurt Gödel's beliefs regarding the power of the continuum and a misleading characterization of Gödel's demise. Gödel in fact believed the size of the continuum to be $\aleph_2$ (*4*, *5*), and rather than dying "in confinement," he lived at home until the last two weeks of his life and coherently discussed mathematical philosophy until the end (*6*).

It's disappointing that the publisher has forgone fact-checking this complex book. Even more discouraging, *Everything and More* is the first in a projected Norton "Great Discoveries" series of popular science books. Caveat emptor.

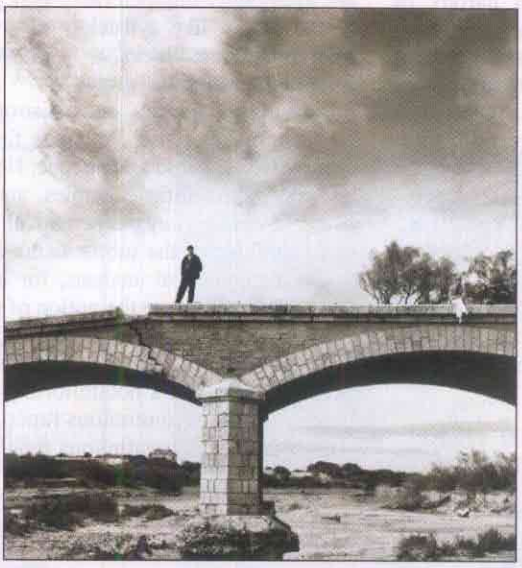
References

1. D. F. Wallace, *Infinite Jest* (Little, Brown, Boston, 1996).
2. G. Plimpton, *Out of My League* (Harper, New York, 1961).
3. G. Plimpton, *Paper Lion* (Harper and Row, New York, 1966).
4. K. Gödel, in *Kurt Gödel, Collected Works, vol. III: Unpublished Essays and Lectures*, S. Feferman *et al.*, Eds. (Oxford Univ. Press, Oxford, 1995), pp. 420–422.
5. W. H. Woodin, *Not. Am. Soc. Math.* **48**, 567 (2001).
6. J. W. Dawson, *Logical Dilemmas: The Life and Work of Kurt Gödel* (A. K. Peters, Wellesley, MA, 1997).

NOTA BENE: MATERIALS SCIENCE

Failure in Many Forms

A university classmate of mine once related that as a youngster he thought he could solve the world's energy crises by extracting fuel from excrement. This idea, which came from a mistaken understanding of how farmers were heating their barns from the farm's waste material, led to some interesting distillation experiments, which succeeded only in creating overpowering smells. With a similar sort of curiosity, Mark Eberhart developed a childhood fascination in trying to understand why things break. His enthusiasm also led to some interesting experiments, and it became a guiding force in his life. However, unlike much of the research in the field, which worries about when materials and structures fail and how to prevent them from doing so, Eberhart (now a chemistry professor at the Colorado School of Mines) continued to be interested in the why and how.



Why Things Break Understanding the World by the Way It Comes Apart
by Mark E. Eberhart

Harmony Books, New York, 2003. 271 pp. $24, C$36. ISBN 1-4000-4760-9.

Using a mixture of personal and newsworthy examples, Eberhart opens the book with a couple of chapters outlining the historical development of less breakable materials, most of which were obtained through trial and error. These chapters also introduce a number of key materials science terms, such as work hardening, polycrystalline, and dislocations. The author keeps most of his descriptions simple enough to be understood by the novice reader, and he should be commended for his effective use of analogies. His decision not to include any diagrams seems questionable though, because many of the concepts are inherently visual in nature.

In subsequent chapters, we follow Eberhart to the Massachusetts Institute of Technology and his surprise introductions to Boston in wintertime and to the materials science department where he studied. As through much of the book, the narrative follows the topics that arise due to specific events. Thus we learn about the origins of the department and the central role of John Slater in its formation, as well as metal embrittlement and its role in sinking the Titanic and numerous warships. While Eberhart explores these topics, he also provides his personal views on the Challenger explosion, the automobile industry, and his experiences as an expert witness. This is perhaps the book's greatest strength, as the author clearly cannot be typecast as a nerdy scientist.

By the end of *Why Things Break*, readers will have learned only a little about what makes materials come apart and far more about what has been done to prevent them from doing so. In addition to discussing basic scientific ideas, Eberhart also reveals his views on the breakdowns in the use of scientific terms, the failures of funding bodies when dealing with new research areas, and the difficulties in getting society to recognize the compromises among safety, reliability, cost, and the need for all objects to fail at some stage in their lifetime. Although some of these tangential topics are unnecessary distractions, they do add to an understanding of Eberhart as a scientist, and many are worthy of broader discussions within society.

And what became of my classmate? Much like Eberhart, he followed an interesting path in the world of engineering. Sensibly, he got out of the energy business (his mother forbade any more experiments along those lines). Instead, he now heads a small biomaterials company that makes products for tissue regeneration, functional tissue replacements, and drug delivery. Which only goes to show that one thing you should never break is the curiosity of a child.

–MARC LAVINE

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