3/8/72 4:20 – 5:50 I.A.S. Conversation with Kurt Gödel.

① Our intuition that a nowhere dense set is like a point and hence the Strong Baire Category Theorem holds is false. This intuition is related to conceiving of the continuum as a set of points – such a consideration leading e.g. to the feeling that $\mathbb{R}$ has gaps [believed by Brouwer (curiously, Takeuti mentioned this belief of Brouwer to me at lunch)]. That this intuition is false is demonstrated by the Hahn-Tarski sphere dissection.

② G. firmly believes in the square axioms, reasoning that diagonalization on countable sets should exhaust the sizes of functions. He suggests

i) The square axioms do imply $2^{\aleph_0} \leq \aleph_2$.

ii) This can be proved by showing that the square axioms imply any decreasing sequence of $F_{\aleph_1}$ [can we show this for F_0 over $\mathbb{N}$ sets has cofinality ω_1. We would use this fact in place of the Borel zero-set measure, not to say that this measure is uninteresting.

iii) The square axioms should have other consequences in Sierpinski-type descriptive set theory. For one thing, the $\aleph_0$ square axiom implies $\mathbb{R}$ is the union of $\aleph_1$ nowhere dense sets. Does not this axiom also imply that $\mathbb{R}$ is the union of $\aleph_1$ measure zero sets?

iv) $\mathbb{R}$ is the union of $\aleph_1$ nowhere dense sets seems

highly desirable to Q. He accepts the falsity of MA for the reason mentioned in (I). He would be interested to see MA reduced to a statement about $\mathbb{R}$.

v) He no longer believes that a Pantachie P would have no $\omega_1 - \omega_1^*$ gaps. In view of the $\aleph_0$-squares axiom we would expect every segment in P to have ordinality ω_1, but should expect P to be more like the rationals than the reals, thus allowing the $\omega_1 - \omega_1^*$ gaps. Says he only used this axiom in his paper to have some reason for claiming $\aleph_1 < 2^{\aleph_0}$. [Adds that he was ill and taking strong drugs at the time he wrote this

paper. Takeuti even implied that he goaded G. into writing this paper. Does G. hold this against Takeuti? He appears to be ignorant of or uninterested in Nodal Transfinite Type Theory [I was discussing NTT with Takeuti today. He said I was the only person who had ever gotten into the philosophical implications of it]].

vi) G. believes that a major result is in fact a uniform system of cardinal notation, as I once claimed.

vii) G. feels the Aquaro axiom should hold at each regular cardinal.

③ In justifying axioms of Infinity, G. takes the ω-doppelgangers approach — reasoning that the particularly simple properties such as measurability should exist. This seemed to be his only justification for MC; he seemed rather uninterested, nay incredulous, of steering Our research with NTT. I weighed indulgently at my grandiose projections.

④ I asked, why is $V \neq L$? "We believe in measurable cardinals", he said. Then continued with the following more intuitive reason: To show $V \neq L$ we will show $\aleph_1^{(L)}$ is countable. Now if we restrict ourselves to elements of $R_{\omega+1}$, the

appropriate constructibility hierarchy terminates at some countable ordinal. In general, for any definable type α, the hierarchy for R_α terminates at some countable ordinal. Now in L, to be sure, we insert all the ordinals yet it seems plausible that just as in the other cases, every set of integers in L is constructed before some countable ordinal.

(5) So what of the possibility that every set is countable? He answered: "This is an interesting description of the way we think — i.e. since at any stage we can only branch finitely many times, we can quite possibly

only conceive of countable sets — however, "I do objective mathematics" [as opposed to subjective mathematics]."

A g. indicated that there should be a simpler proof than Kunen's of $Con(ZF+MC) \to Con(ZF+MC+V=K)$.

I should try to produce one using $SM(ZF+MC) \to SM^{(L)}(ZF+MC)$.

Say $L_\alpha[x] \models ZF+MC$, $x \in L$.

Say α is minimal for this property. Say x is minimal in the well-ordering of L...

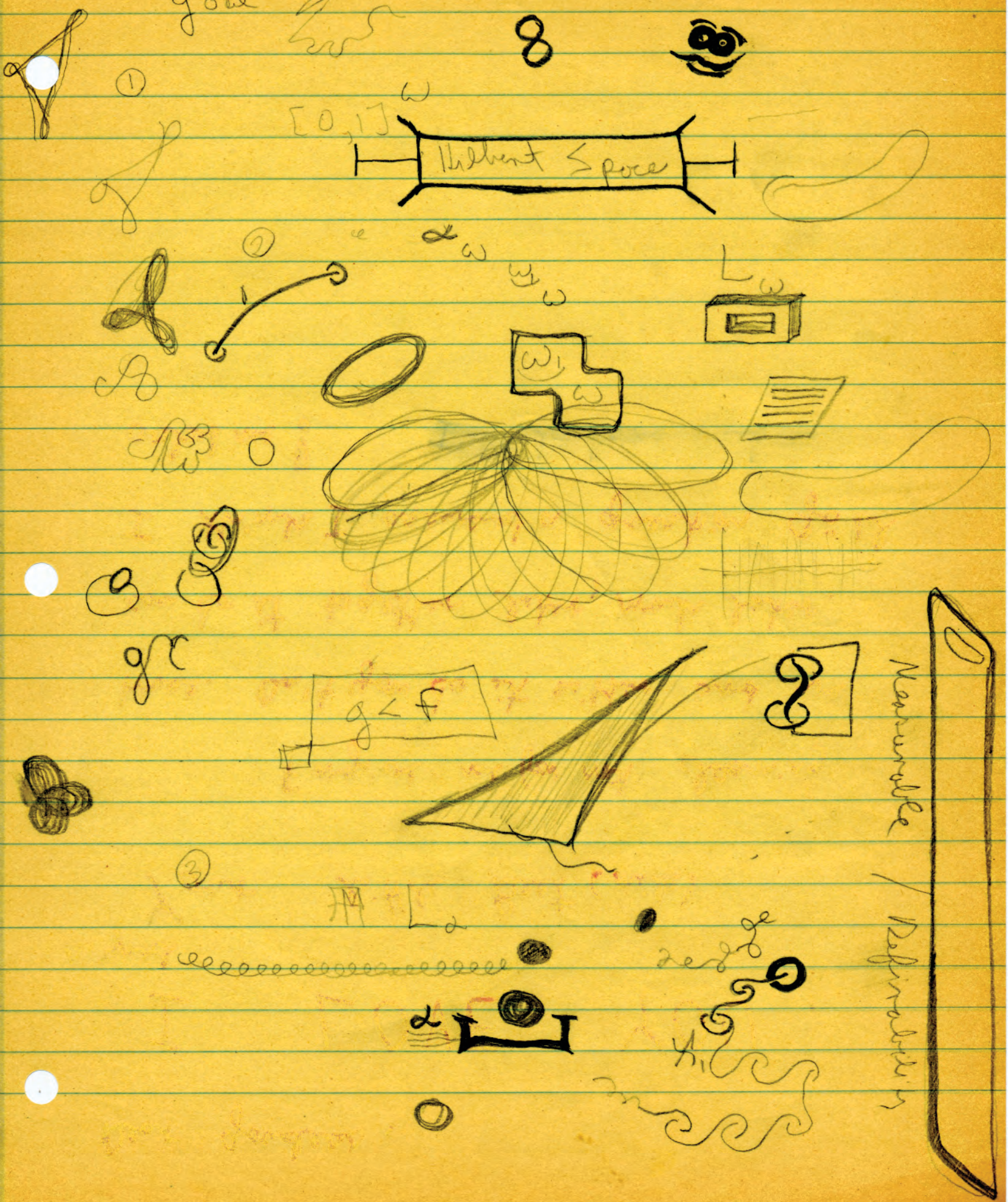
B Can we prove $(\exists f \subseteq P\omega)(\forall \sigma)[\sigma^{\omega}/f$ is saturated$]$ using the $\aleph_0$ square axiom??

C What about definability in the absence of $V=K$? If Θ is inaccessible [i.e. $\Theta \notin V'_{R_\Theta}$. (Note that if $V=L$, Θ a limit cardinal $\rightarrow$ ($\Theta \notin V'_{R_\Theta} \leftrightarrow \Theta \notin V'_{L_\Theta} \leftrightarrow \Theta \notin V'_\Theta$

Q: When does $R_\alpha \models V=K$? Should be quite often esp. since if $R_\alpha \not\models ZF$ then $R_\alpha \models V=K \rightarrow$ there is a definable well-ordering of R_α. So just because there is no definable well ordering of the continuum does not imply that we cannot have $R_\alpha \models V=K$ at certain limit ordinals α.

Phone Conversation with K. Gödel 3/10/72 Zeno's

gödel



6/19/72

Called Gödel (609-924-0569).

He said I should go to Genoa, come see him in late July, maybe come to IAS next year.

He liked techniques of Natural Scales.

Concerning my conception of axiom of infinity he said "This only answers the problem for a certain finitary language. L is essentially an infinitary language." Or he said "I wasn't as you characterize the universe as L[ω] you have not answered the question, since the use of L involves 'knowing what infinity means already'" or he said something else which I couldn't catch up with.

I asked him if the paradoxes of time travel could be dealt with. He said they could, but not in a way he could explain in a few minutes. He said these paradoxes (I suggested "what if you killed your father" in particular) were not essential like Russel's and Epimenes' but were merely the result of carelessness.

I.A.S. August 7, 1972 - Conversation with Gödel - 4:30-6:00

① G says ¬CH because of the counter-intuitive consequences of CH given in What is ... .

[Points out that we must be careful about intuition - e.g. Seemingly impossible space-filling curve is continuous but nowhere differentiable, i.e. it vibrates infinitely often.] Says ¬GCH because GCH → $\aleph_\omega$ = product of cardinals is simply the first cardinal > all the cardinals in the product. This is too simple.

② As regards the reality of the objects of mathematical intuition, G. suggests that the world of possibilities has an objective (i.e. not

subject to the will) existence. That possibility is merely a weaker form of reality.

(3) A part of the continuum must be an interval, not a point. The continuum is not made up of points. If we cut a line we do not have to decide whether to put the point where we cut to the right or the left. This point does not exist.

(4) He appeared before his outside glass door, when I least expected, like a fish.

(5) He was excited about my work on Infinity, I gave him the only copy of a little paper showing the incompleteness of ZFInf. He

liked the $(\exists \alpha)[R_\alpha FO]$ characterization of Axiom of Infinity.

⑥ It wonders why measurable cardinal always means 2-valued measurable. Why not $\lambda(\kappa)$-valued, depending on the κ?

⑦ Compares the process of adding all the true axioms of infinity to trying to complete P. We know that in $L_{\omega_1^{CK}}$ we can complete P — something to do with $L_{\omega_1^{CK}}$ being a β-model for recursive sets.

Idea is to get a notion of natural well ordering not requiring all this ω_1^{CK}.

Ash Tokanti.

(2) Claims that if I could travel back in time and kill myself then the theory of relativity would be violated — hence such an act cannot take place. There is no physical reason — it is just that it is impossible that this happen — e.g. anyone who gets to travel in time is too straight [Buzz, Neil] to try it.

He could see, but did not really accept my evaluation of a second dimension to time

(3) "That is really a strange idea" he said to my "tenet" theory [i.e. Everything is inside everything else].

(10) Our discussion of personality grew heated — Suddenly he looked very unhealthy, old. He maintains that we have a personality - charges being produced from the outside, not the inside: It is my contention that everything is outside, that the inside is just "I Am".

(11) V=K should hold, e.g. boys, because the existence of chaos in the world of thought is contradictory. Must then every set be thinkable? Not by Man, he said.

I objected that $K^{(K)} \neq K$ in general (If this were true we could see that reason comes K out of chaos and that $V=K^{(K)}$ holds) —

he responded that some principle, evident (as opposed to V=K) would arise to imply V=K. He seemed optimistic about improving our axiomatization of ¶ and insists that we can think of ¶ Σ rather than some large (arbitrarily large) R_α J.

(12) He seemed weaker than last time I saw him — perhaps I have learned more from my fantasies of G. than G. himself

(13) g., in declaring that you could not kill yourself yesterday, illustrated: "It should be possible to form a complete theory of human behaviour — i.e. predicting from the hereditary + environmental givens what a person will do. However, if a mischevious person learns of this theory he can act in a way so as to negate it. Hence I conclude that such a theory exists but That no mischevious person will learn of it — In the same way time travel is possible, but no person will ever manage to kill his past self.

July 1, 1975

Called Godel at home.

He s-aid that my recent research was "tending in very bizarre directions", and that i wouldn't get into the Institue as long as i ketp it up. I might get in if i went back to the type of think i did on my thesis.

"set theory is an idealization, and the power-set axiom is one of the first necessary idealizations."
"any axiom of infinity which decided the continuum problem will do so on the basis of the "orders of growth" axioms."

I told him i was thinking of giving up mathematics to write science-fiction. He was horrified, but laughing, urged me to go into the foundations of physics instead.

He said i was right in saying that there is n omeaning to statements about the class of all sets. His solution was to have the concept of all sets as an intensional, non-extensional, thing.

I asked what concepts were, what about the con cept of all concepts. He responded that concepts need not be well-founded. Of course, he added, olne could speak of well-founded concepts, as,ing e.g. if the concept of all well-founded conepts is well-founded."But,"he added,"there is no theory of concepts".

He caustioned against blending the metaphysical Absolute with the concept of all well-founded sets. "It is provable that you get all the sets by iteration the power set operation"

"I cannot believe that your bizarre methods will lead anywhere."

I got snotty and said "Wll, that's a chance you have to take" first i said "well, you know, i still think my ideas are correct" and then he said something like "But no one in the whole world agrees with you" and then the first thing.

earliet i argued that sets are not just idealization, that set theory should be tied to something real."there you are thinking of nameable sets" he said.

March 28, 1977

Threw the Ching, got six yang lines with a nine in second place. Called Godel's number, got the wrong number, called again, got his wife who said call again at 4.
Called at 4, got godel, said he was eating lunch, call again at 4;30.
called at 4:30, say I would like to talk to him about some things.
"Talk about what?" he asks, suspiciously, perhaps expecting me to ask for smoething or argue about set theroy.

I asked him about Q.M., did he agree with Einstein that the copenhagen interpretaion was just a positivistic theory expressing the limit of human knowledge rather than a realisti c theory of how the world is. Yes, he agreed.

Why is the future not predictiable then, though I asked, does this mean that time branches? I tried to sketch the MU interpretation, which he seemed not to be familiar with. I came back to this again later. The gist of what he said was that he did believe that there could easily be other universse which had no coausal link at all with ours...that it would be rather strange if there wre just this one finite universe with its peculiarities [He said, also, that he did not think that our universe had any actual physcial infinities. When I asked about if there could not beinfinities in the small beyond the wuantum mlim it he seemed to say that there could be, but not for us to see]. He did not think it reasonable that <u>every</u> possible universe exists.

Why does the I Ching work,I asked? He said that the world has <u>structural</u> properties as well as <u>causal</u> properties, which physics studies.

I asked about the Mind. That is, iw repaeated his remarks that there could be inprinciple a machine which duplicates us, and would there be a Mind beyond theis <u>behaviour</u>? He said yes, the Mind is the thing which is structured but it exists independently of the individual properties. I then asked if he beleived that the Mind is everywhere, One. Of course, he says, this is the basic mystic teaching. I was happy to see that without my suggesting it he did refer to the classical mystic tradition.

I asked , "What causes the illusion of the passage of time?", after referring to his paper as I used it in my book. He spoke not directly to this question, but to the question of what my question meant, that is wh-I would even believe that there is a perceived passage of time (which he practically denied, saying that this position is to be wrong. He related the getting rid of the passage of time to the mystic

ONe Mind which was referred to r.e. the question about mechanical intelligence) With regard to the question about the passage of time he said that this arises from the confusing of the given with a reality. Passage of time arises becasue we think of occupying different realities. In fact, we occupy only different givens. There ios olny one reality.
He also remarked, on this point, that we seem to be more interested in the future than the past.

I asked if the unpredictability of the future did not lead to branching time perhaps. He came back to his old contention that the future is in fact predictable. But, I asked, if you can find out how to see the future, aan't you then falsify the technique by deliberately doing something other than that which wqs predicted? He ansered that if you intended to do that, then you would not learn the prediction scheme. A great answer. He clatified that you do not go and do what you can see that you are poing to do, except perhaps there is , he granted, unpredictability in verysmall actions. "Which shoe do you put on first" was the example I used. He seemed to say, though I didn't quite hear, "But why wear shoes?"

It was a pleasant enough conversation. At one point he said,"this converstaion is not leading anywahere", but then it picked up again. AT the end he was friendly and seemed interested to see my book, which I said I'd send.

I asked several times about the many universes notion, he said it would take a lot of work to see what was behind my questions.